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Kelvin-Helmholtz instability in the polar cusp region of the magnetosphere

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Abstract Shear driven Kelvin-Helmholtz instability in the polar cusp region of magnetosphere (adjacent to ionosphere) for a low β-plasma has been studied. With the help of suitable dispersion relation, the dispersion characteristic and growth rate of Kelvin-Helmholtz waves (in acoustic mode waves) for different shear strengths have been calculated and the results have been used to explain ULF magnetic noise observed in the polar cusp region of the magnetosphere Thus, the wave turbulence generated by this instability may be responsible for alternative electron pitch angle

Keywords Kelvin-Helmholtz instability, polar cusp region, magnetosphere (ionosphere)

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One of the most important sources which controls the magnetospheric and ionospheric phenomena is the solar wind interaction with the Earth's magnetosphere. Due to this interaction, Earth's magnetopause boundary becomes highly unstable to the Kelvin-Helmholtz instability and it has been used extensively to explain various obsevred magnetospheric and ionospheric phenomena [1-6].

The aim of the present note is to study K-H instability for a finite β-plasma and to explain ULF magnetic noise observed in the polar cusp region adjacent to ionosphere. A suitable expression for dispersion relation has been considered and hence the growth rate of this instability has been analysed.

The coordinate system for the plasma parameters has been shown in Figure 1. This model consists of a plasma (collisionless) flow (electrons and ions) parallel to an uniform magnetic field $\boldsymbol{B}_0$ (along z-axis) with a velocity v_0 and varying spatially in a direction perpendicular to it (*i.e.* along X-axis). Inhomogeneity in the plasma density has been considered along X-axis. Thus, the gradients of both velocity v_0 and density n_0 are taken in X-direction, respectively. The diamagnetic drift velocity (v_D) is perpendicular to $\boldsymbol{B}_0$. The displayed geometry

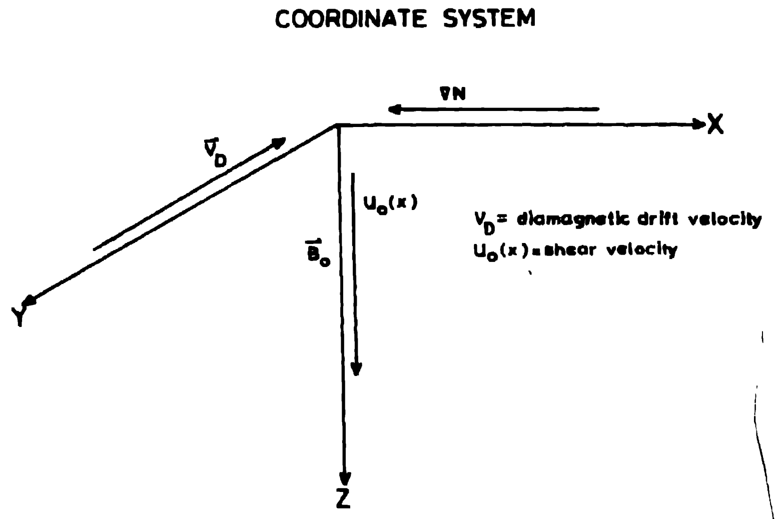

Figure 1. The coordinate system for plasma parameters

in Figure 1 for plasma parameters conforms the polar cusp region with solar wind plasma flowing into the ionosphere along the Earth's magnetic field lines. The dispersion relation for arbitrary β for K-H instability can be written as [7] ·

$$\left[1+\frac{1}{2}\beta_i\frac{k_y}{k_z}\xi-\frac{\Omega_i^2}{k_z^2v_A^2}-\frac{\omega_i^*\Omega_i}{2k_z^2v_A^2}(1+\beta)\right]\times\left\{\frac{k_z^2v_i^2}{\Omega_i^2}\left(1-\frac{k_y}{k_z}\xi\right)\right.$$

$$\left.\times\left(1+\frac{T_e}{T_i}+\Omega_i\frac{\gamma_e-1}{\Omega_e^\gamma}\frac{T_e}{T_i}\right)+\left(1+\frac{\omega_i^*+|\omega_e^*|}{\Omega_i}\right)\left[\Omega_i\Gamma-(1+\beta)\right]\right\}=0, \quad (1)$$

where

$$\Gamma=\frac{\gamma_i-1}{\Omega_i^\gamma}\left[-\beta_i-\beta\frac{\omega_i^*}{\Omega_i}+\frac{k_z^2v_i^2}{\Omega_i^2}\left(1-\frac{k_y}{k_z}\xi\right)\right]-\beta_e\frac{\gamma_e-1}{\Omega_e^\gamma},$$

$$\beta=\frac{4\pi n_0(T_{i0}+T_{e0})}{B_0^2},\ \beta_{i,e}=\frac{4\pi n_0T_{i,e}}{B_0^2},$$

$\Omega_{i,e}=\omega-k_zv_0-k_yv_{Di,e}$ are frequencies Doppler shifted by the plasma flow along the magnetic field (k_z) and by the diamagnetic drift perpendicular (k_y) to it. k_y = wave number perpendicular to $\boldsymbol{B}_0$, k_z = wave number parallel to $\boldsymbol{B}_0$. $v_{Di,e}=(k_BT_{i,e}/q_{i,e}B_0)(n_0'/n_0)$, ion and electron diamagnetic drift. k_B is Boltzmann constant. Here, the drift frequency has been introduced as $\omega_{i,e}^*=k_y(k_BT_{i,e}/q_{i,e}B_0)(n_0'/n_0)$, and the abbreviated notation

$$\Omega_{i,e}^\gamma=\Omega_{i,e}+\omega_{i,e}^*(1+\gamma_{i,e}\beta)$$

and γ is particle specific heat.

Gradient of both density (n_0') and velocity (v_0') are taken in the X-direction. The strength of the velocity shear in the successive stability analysis is

$$\xi = v_0' / \Omega_{ci},$$

where, $\Omega_{ci} = q_i B_0 / m_i$, ion-cyclotron frequency.

and $v_A^2 = B_0^2 / 4\pi n_0 m_i$, Alfvén wave speed.

The above eq. (1) has two factors : first factor on the left hand side deals about Alfvén wave propagation along magnetic field, while second one deals about slow magnetosonic wave (ion-acoustic mode wave). The plasma conditions under consideration, fulfil the magnetosonic wave. The lower vlaue of β makes the wave harder to be excited for a given velocity shear. However, the magnetosonic wave mode (ion-acoustic like mode) has been found to be more unstable for low β contrary to Alfvén wave mode. In the presence of the density inhomogeneity, an analytical treatment of the magnetosonic branch of the dispersion relation is possible only in the isothermal case (= $\gamma_e = \gamma_i = \gamma = 1$). In the absence of density inhomogeneity, the finite β affects only the magnitude of the frequency but not the instability threshold.

The solutions of the magnetosonic wave (ion-acoustic mode wave) are written from eq. (1) as

$$\omega = \frac{1}{2}\left\{-\left[\omega_i^*\left(\frac{T_e}{T_i}-1\right)-2k_z v_0\right] \pm \sqrt{\left(1+\frac{T_e}{T_i}\right)^2 \omega_i^{*2} + \frac{4}{1+\beta} k_z^2 c_s^2 \left(1-\frac{k_y}{k_z}\xi\right)}\right\}, \quad (2)$$

where $c_s^2 = (T_i + T_e) / m_i$ and ω is complex frequency (ω+ Re ω+ Im ω).

The instability conditions are

$$(k_y / k_z)\xi > 0, \text{ and } \frac{k_y}{k_z}\xi > 1 + \frac{1}{4}\left(1 + \frac{T_e}{T_i}\right)(1+\beta)\frac{k_y^2}{k_z^2}\frac{\alpha_i^2}{L_N^2},$$

$\alpha_i = v_{Thi} / \Omega_{ci}$, being the ion Larmer radius, L_N is typical inhomogeneity scale length. The eq. (2) can be further expressed as

$$\text{Re}\,\omega = \omega_r = k_z v_0\left[1 - \frac{1}{2}\left(1 - \frac{T_e}{T_i}\right)\frac{v_i}{v_0}\frac{k_y}{k_z}\frac{\alpha_i}{L_N}\right] \quad (3)$$

and

$$\text{Im}\,\omega = \omega_i = k_z c_s \sqrt{\frac{1}{(1+\beta)}\left[\frac{k_y}{k_z}\xi - \left\{1 + \frac{1}{4}\left(1 + \frac{T_e}{T_i}\right)(1+\beta)\frac{k_y^2}{k_z^2}\frac{\alpha_i^2}{L_N^2}\right\}\right]}. \quad (4)$$

The above eqs. (3) and (4) are the required expressions for the wave characteristics (Re $\omega = \omega_r$) and growth rate γ(Im $\omega = \omega_i$) of instability respectively. For $\beta = 0$ the eq. (4) reduces to electrostatic analysis of K-H instability [8]. For the plasma model under consideration, in the polar cusp region, $\beta < 1$ condition is fulfilled.

The considered plasma parameters are taken from Dobrowolny [7] : $\beta = 0.6$, $T_e/T_i = 0.01$, $\alpha_i/L_N = 0.05$, the observed proton density = 20/cm^3, thermal velocity of the ions $v_{Thi} = 2.4 \times 10^7$ cm/sec., streaming velocity along the geomagnetic field lines $v_o = 2 \times 10^7$ cm/sec., ion-cyclotron frequency $\Omega_{ci} = 200$ rad/sec. and $k_y = 10^{-6}$ cm^{-1}. With the help of the above plasma parameters, the variations in dispersion feature and the growth rate with k_z/k_y ratio for K-H instability have been computed and these results have been displayed in Figure 2. The growth rate first increases with increase of k_z/k_y and attains a maximum. Afterwards, it falls sharply. The growth rate has been found to increase with increase of velocity shear (ξ). The unstable wave frequency does not depend on velocity shear (ξ). The increase of unstable wave frequency is associated only with the increase of k_z/k_y ratio. For $k_z/k_y < 10^{-2}$ the diamagnetic drift dominates and is responsible for the growth of the drift waves (diamagnetic). The unstable wave frequencies are between 0.2 Hz to 20 Hz. Thus, with the help of simple model calculation, an attempt has been made to show that the K-H instability can also be excited in the polar cusp region at high altitudes adjacent to ionosphere. A more appropriate plasma model (with more plasma parameter conditions in it) will further provide the effect of K-H instability on charged particle characteristics and several ULF-ELF magnetic noise at magnetospheric height in the polar cusp region.

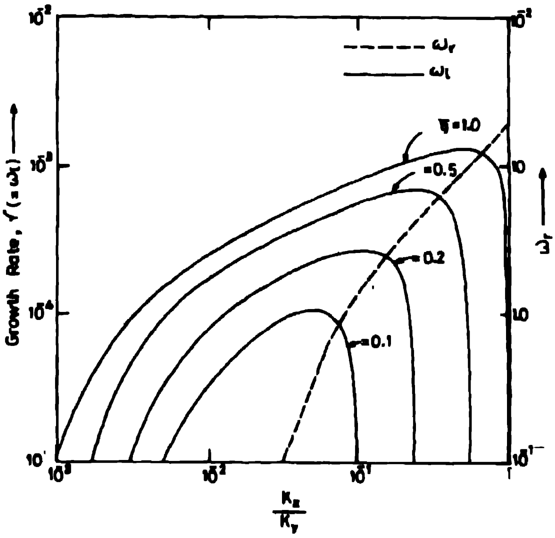

Figure 2. Variation of the imaginary part of frequency Im ω, growth rate γ(ω_i-solid lines) and real part of frequency Re ω (ω_r-dashed line) with parallel wave length (k_z/k_y) for $k_y = 10^{-6}$ cm^{-1}, $T_e/T_i = 0.01$; $\alpha_i/L_N = 0.05$; $\beta = 0.6$; shear velocity $v_{Thi} = 2.4 \times 10^7$ cm/sec ; $\omega_p = 4.2 \times 10^1$ rad/sec, $\omega_{ci} = 200$ rad/sec and $\xi = 0.1$, 0.2, 0.5, and 1 0

Hawkeye-1 spacecraft has observed a variety of plasma wave motions in the high altitude of polar cusp region [6, 9, 10]. We are interested only in ULF-ELF magnetic noise. Usually, the ULF-ELF magnetic noise is more intense in the polar cusp ; however, the intensity often varies gradually from the polar cusp into the magnetosheath [6]. At magnetospheric height this magnetic noise has been regarded as turbulence, generated mainly by the Kelvin-Helmholtz instability [10]. This turbulence had already been interpreted in terms of steady state noise spectrum at high altitudes of the polar cusp. Atmospheric explorer (AE-C) satellite have enabled detailed characteristics of the cusp to be measured at low altitudes (282km-approximately, 71° invariant latitude). AE-C spacecraft does not include wave experiments as Hawkeye-1 does. It provides the energy particle characteristics and magnetospheric electron temperatures. Even in the absence of wave experiments, the high magnetospheric electron temperature and unusual low energy electron pitch angle distributions may be attributed to the Kelvin-Helmholtz instability, driven by the shear between the proton flow down along the magnetic lines of field and the ambient magnetosphere [10]. In this work, it has been found that the unstable wave frequencies are from 0.2 Hz to 20 Hz which lies well within the observed ULF-ELF magnetic noise.

The further extension of this work will be to include ion inertia effect [11], temporally and spatially varying cold plasma effect [12] along with thickness of the shear layer which are very important in the polar cusp region. The ion inertia effect plays important role in catering the energetic particles and mixing the plasma in polar cusp region. The cold plasma effect may be helpful for deciphering the complex processes involved in ionosphere-magnetosphere coupling. A detailed study of K-H instability for different physical conditions prevailing in the medium is very much needed as K-H instability does not only play an inportant role in Earth's atmosphere but also seems to play an important role : (i) at the venus magnetopause, a candidate process for the formation of flux ropes [13] and for viscous drag force to transport the magnetospheric plasma from the day side to the night side [14], (ii) at the Sun surface, controlling the rising flux tubes (magnetic features on the sun) in the convective zone [15].

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